

**Class X Session 2025-26**  
**Subject - Mathematics (Standard)**  
**Sample Question Paper - 06**

**Time Allowed: 3 hours**

**Maximum Marks: 80**

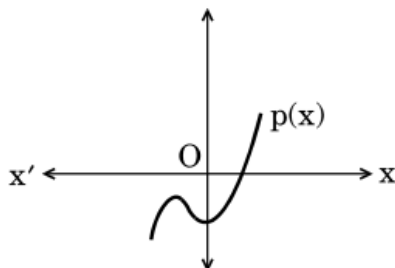
### General Instructions:

Read the following instructions carefully and follow them:

1. This question paper contains 38 questions.
2. This Question Paper is divided into 5 Sections A, B, C, D and E.
3. In Section A, Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.
4. In Section B, Questions no. 21-25 are very short answer (VSA) type questions, carrying 02 marks each.
5. In Section C, Questions no. 26-31 are short answer (SA) type questions, carrying 03 marks each.
6. In Section D, Questions no. 32-35 are long answer (LA) type questions, carrying 05 marks each.
7. In Section E, Questions no. 36-38 are case study-based questions carrying 4 marks each with sub-parts of the values of 1, 1 and 2 marks each respectively.
8. All Questions are compulsory. However, an internal choice in 2 Questions of Section B, 2 Questions of Section C and 2 Questions of Section D has been provided. An internal choice has been provided in all the 2 marks questions of Section E.
9. Draw neat and clean figures wherever required.
10. Take  $\pi = \frac{22}{7}$  wherever required if not stated.
11. Use of calculators is not allowed.

## Section A

1. The prime factorisation of 432 is: **[1]**
- a)  $2^4 \times 3^3$  b)  $2^3 \times 3^4$
- c)  $2^3 \times 3^3$  d)  $2^4 \times 3^4$
2. Number of zeroes of the polynomial  $p(x)$  shown in the Figure, are: **[1]**



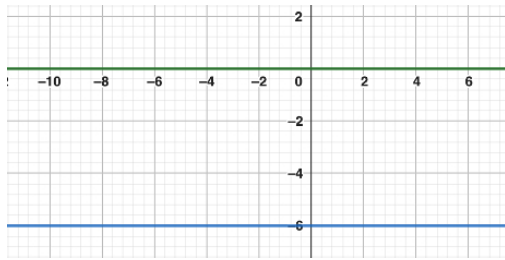
- a) 2 b) 3

c) 0

d) 1

3. The pair of linear equations  $y = 0$  and  $y = -6$  has:

[1]



a) no solution

b) only solution (0, 0)

c) a unique solution

d) infinitely many solutions

4. The roots of the quadratic equation  $x^2 - 4 = 0$  is/are:

[1]

a) 2 only

b) -2, 2

c) -4, 4

d) 4 only

5. The  $n$ th term of an AP is  $7 - 4n$ , then its common difference is

[1]

a) -3

b) 4

c) -4

d) 3

6. The distance between the points  $P\left(-\frac{11}{3}, 5\right)$  and  $Q\left(-\frac{2}{3}, 5\right)$  is:

[1]

a) 3 units

b) 4 units

c) 2 units

d) 6 units

7. The distance between the points (0, 0) and (a - b, a + b) is

[1]

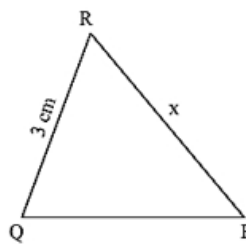
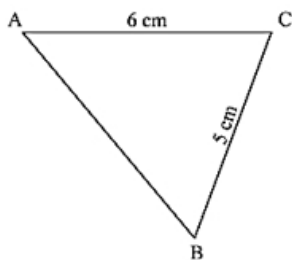
a)  $2\sqrt{ab}$

b)  $\sqrt{2a^2 + ab}$

c)  $2\sqrt{a^2 + b^2}$

d)  $\sqrt{2a^2 + 2b^2}$

8.



[1]

In the given figure,  $\triangle ABC \sim \triangle QPR$ . If  $AC = 6$  cm,  $BC = 5$  cm,  $QR = 3$  cm and  $PR = x$ ; then the value of  $x$  is:

a) 3.6 cm

b) 2.5 cm

c) 3.2 cm

d) 10 cm

9. In Figure, a circle touches the side  $DF$  of  $\triangle EDF$  at  $H$  and touches  $ED$  and  $EF$  produced at  $K$  and  $M$  respectively. If  $EK = 9$  cm, then the perimeter of  $\triangle EDF$  is

[1]



- 
- Figure 1 shows a circle with center  $O$ . A vertical chord  $AB$  is drawn. Two tangent lines from an external point  $P$  touch the circle at  $A$  and  $B$ . The angle  $\angle APB$  is labeled as  $80^\circ$ .

-  [www.studentbro.in](http://www.studentbro.in)

a)  $P(A) < 0$

b)  $0 \leq P(A) \leq 1$

c)  $-1 \leq P(A) \leq 1$

d)  $P(A) > 1$

17. A dice is thrown once. The probability of getting an odd number is

[1]

a)  $\frac{2}{6}$

b)  $\frac{4}{6}$

c) 1

d)  $\frac{1}{2}$

18. For the following distribution:

[1]

Marks Below	10	20	30	40	50	60
Number of Students	3	12	27	57	75	80

the modal class is:

a) 20 – 30

b) 40 – 50

c) 50 – 60

d) 30 – 40

19. **Assertion (A):** A sphere of radius 7 cm is mounted on the solid cone of radius 6 cm and height 8 cm. The volume of the combined solid is  $1737.97 \text{ cm}^3$ .

[1]

**Reason (R):** Volume of sphere is  $\frac{4}{3}\pi r^3$ .

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** The constant difference between any two terms of an AP is commonly known as common difference.

[1]

**Reason (R):** The common difference of 2, 4, 6, 8 this A.P. is 2.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

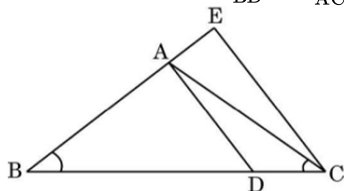
### Section B

21. Prove that  $5 + 2\sqrt{3}$  is an irrational number, given that  $\sqrt{3}$  is an irrational number.

[2]

22. In the given figure,  $\frac{BC}{BD} = \frac{BE}{AC}$  and  $\angle ABD = \angle ACD$ . Show that  $\triangle ABD \sim \triangle EBC$ .

[2]



23. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

[2]

24. Evaluate:  $2(\sin^2 45^\circ + \cot^2 30^\circ) - 6(\cos^2 45^\circ - \tan^2 30^\circ)$

[2]

OR

Prove that  $\frac{\cot A - \cos A}{\cot A + \cos A} = \sec^2 A + \tan^2 A - 2 \sec A \tan A$ .

25. Find the area of the minor and the major sectors of a circle with radius 6 cm, if the angle subtended by the minor arc at the centre is  $60^\circ$ . (Use  $\pi = 3.14$ )

[2]

OR

An arc of a circle of radius 10 cm subtends a right angle at the centre of the circle. Find the area of the corresponding



major sector. (Use  $\pi = 3.14$ )

### Section C

26. 144 cartons of Coke cans and 90 cartons of Pepsi cans are to be stacked in a canteen. If each stack is of the same height and if it equal contain cartons of the same drink, what would be the greatest number of cartons each stack would have? [3]
27. If  $\alpha$  and  $\beta$  are zeroes of the quadratic polynomial  $4x^2 + 4x + 1$ , then form a quadratic polynomial whose zeroes are  $2\alpha$  and  $2\beta$ . [3]
28. Find the sum of the first 15 multiples of 8. [3]

OR

If  $S_n$  denotes, the sum of the first  $n$  terms of an A.P. prove that  $S_{12} = 3(S_8 - S_4)$ .

29. Two concentric circles are of radii 7 cm and  $r$  cm respectively, where  $r > 7$ . A chord of the larger circle of length 46 cm, touches the smaller circle. Find the value of  $r$ . [3]

OR

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that  $\angle PTQ = 2\angle OPQ$ .

30. Prove that  $\frac{\tan \theta}{1 - \tan \theta} - \frac{\cot \theta}{1 - \cot \theta} = \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$ . [3]
31. Find the mean of the following data, using step-deviation method: [3]

Class	5 - 15	15 - 25	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75
Frequency	6	10	16	15	24	8	7

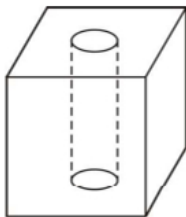
### Section D

32. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the park. [5]

OR

If the roots of the quadratic equation  $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$  are equal. Then show that  $a = b = c$

33. From the top of an 8 m high building, the angle of elevation of the top of a cable tower is  $60^\circ$  and the angle of depression of its foot is  $45^\circ$ . Determine the height of the tower. (Take  $\sqrt{3} = 1.732$ ). [5]
34. In Figure, from a solid cube of side 7 cm, a cylinder of radius 2.1 cm and height 7 cm is scooped out. Find the total surface area of the remaining solid. [5]



OR

A tent is in the shape of a right circular cylinder up to a height of 3 m and then a right circular cone, with a maximum height of 13.5 m above the ground. Calculate the cost of painting the inner side of the tent at the rate of ₹ 2 per square metre, if the radius of the base is 14 m.

35. Find the mode, median and mean for the following data: [5]

Marks Obtained	25 - 35	35 - 45	45 - 55	55 - 65	65 - 75	75 - 85
Number of students	7	31	33	17	11	1



## Section E

36. Read the following text carefully and answer the questions that follow:

[4]



Lokesh, a production manager in Mumbai, hires a taxi everyday to go to his office. The taxi charges in Mumbai consists of a fixed charges together with the charges for the distance covered. His office is at a distance of 10 km from his home. For a distance of 10 km to his office, Lokesh paid ₹ 105. While coming back home, he took another route. He covered a distance of 15 km and the charges paid by him were ₹ 155.

- i. What are the fixed charges? (1)
- ii. What are the charges per km? (1)
- iii. If fixed charges are ₹ 20 and charges per km are ₹ 10, then how much Lokesh have to pay for travelling a distance of 10 km? (2)

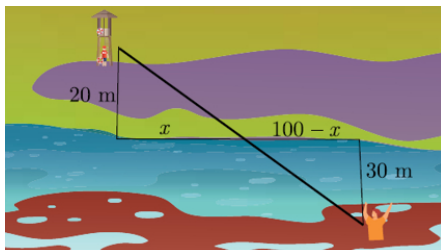
**OR**

Find the total amount paid by Lokesh for travelling 10 km from home to office and 25 km from office to home. [Fixed charges and charges per km are as in (i) & (ii). (2)]

37. Read the following text carefully and answer the questions that follow:

[4]

**Swimmer in Distress:** A lifeguard located 20 metre from the water spots a swimmer in distress. The swimmer is 30 metre from shore and 100 metre east of the lifeguard. Suppose the lifeguard runs and then swims to the swimmer in a direct line, as shown in the figure.



- i. How far east from his original position will he enter the water? (Hint: Find the value of  $x$  in the sketch.) (1)
- ii. Which similarity criterion of triangle is used? (1)
- iii. What is the distance of swimmer from the shore? (2)

**OR**

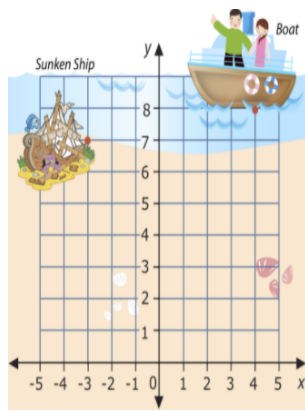
What is the length of AD? (2)

38. Read the following text carefully and answer the questions that follow:

[4]

Mary and John are very excited because they are going to go on a dive to see a sunken ship. The dive is quite shallow which is unusual because most sunken ship dives are found at depths that are too deep for two junior divers. However, this one is at 40 feet, so the two divers can go to see it.





They have the following map to chart their course. John wants to figure out exactly how far the boat will be from the sunken ship. Use the information in this lesson to help John figure out the following.

- What are the coordinates of the boat and the sunken ship respectively? (1)
- How much distance will Mary and John swim through the water from the boat to the sunken ship? (1)
- If each square represents 160 cubic feet of water, how many cubic feet of water will Mary and John swim through from the boat to the sunken ship? (2)

**OR**

If the distance between the points  $(x, -1)$  and  $(3, 2)$  is 5, then what is the value of  $x$ ? (2)

# Solution

## Section A

1. (a)  $2^4 \times 3^3$

**Explanation:**

$$2^4 \times 3^3$$

2.

(d) 1

**Explanation:**

We see that the graph cuts the x-axis at 1 point which implies p(x) is zero at this 1 point only.

3. (a) no solution

**Explanation:**

Since, we have  $y = 0$  and  $y = -6$  are two parallel lines.  
therefore, no solution exists.

4.

(b) -2, 2

**Explanation:**

$$x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

roots are +2, -2

5.

(c) -4

**Explanation:**

Given:  $a_n = 7 - 4n$

$$\therefore a_1 = 7 - 4 \times 1 = 7 - 4 = 3$$

$$a_2 = 7 - 4 \times 2 = 7 - 8 = -1$$

$$\therefore d = -1 - 3 = -4$$

6. (a) 3 units

**Explanation:**

$$PQ = \sqrt{\left(\frac{-2}{3} + \frac{11}{3}\right)^2 + (5 - 5)^2}$$

$$= \sqrt{9 + 0}$$

$$= 3 \text{ units}$$

7.

(d)  $\sqrt{2a^2 + 2b^2}$

**Explanation:**

distance between the point. (0, 0) and (a - b, a + b) is

$$= \sqrt{(a - b - 0)^2 + (a + b - 0)^2}$$

$$= \sqrt{(a - b)^2 + (a + b)^2}$$

$$= \sqrt{a^2 + b^2 - 2ab + a^2 + b^2 + 2ab}$$

$$= \sqrt{2(a^2 + b^2)} = \sqrt{2a^2 + 2b^2} \text{ units.}$$





$$\angle PBA = 50^\circ$$

$$\therefore \angle PAB = 50^\circ$$

11.

(d) -1

**Explanation:**

-1

12.

(c)  $\frac{\sqrt{1+\cot^2 \theta}}{\cot \theta}$

**Explanation:**

As we know that,

$$\sec^2 \theta = 1 + \tan^2 \theta$$

$$\text{and } \cot \theta = \frac{1}{\tan \theta}$$

$$\Rightarrow \tan \theta = \frac{1}{\cot \theta}$$

$$\therefore \sec^2 \theta = 1 + \left( \frac{1}{\cot \theta} \right)^2$$

$$= 1 + \frac{1}{\cot^2 \theta}$$

$$\Rightarrow \sec^2 \theta = \frac{\cot^2 \theta + 1}{\cot^2 \theta}$$

$$\Rightarrow \sec \theta = \frac{\sqrt{1+\cot^2 \theta}}{\cot \theta}$$

13.

(d)  $\frac{20}{\sqrt{3}}$  m

**Explanation:**

$$\frac{20}{\sqrt{3}} \text{ m}$$

14.

(c)  $45^\circ$

**Explanation:**

Given

$$\frac{\text{area of sector}}{\text{area of circle}} = \frac{1}{8}$$

$$\frac{\frac{\theta}{360^\circ} \times \pi r^2}{\pi r^2} = \frac{1}{8}$$

$$\frac{\theta}{360^\circ} = \frac{1}{8}$$

$$\theta = \frac{360^\circ}{8}$$

$$\theta = 45^\circ$$

15.

(c)  $72^\circ$

**Explanation:**

It is given that area of the sector =  $69.3 \text{ cm}^2$

and Radius = 10.5 cm

$$\text{Now, Area of the sector} = \frac{\pi r^2 \theta}{360}$$

$$\Rightarrow \frac{\pi \times (10.5)^2 \times \theta}{360} = 69.3$$

$$\Rightarrow \theta = \frac{69.3 \times 360 \times 7}{10.5 \times 10.5 \times 22} = 72^\circ$$

Therefore, Central angle of the sector =  $72^\circ$

16.

(b)  $0 \leq P(A) \leq 1$

**Explanation:**

The probability of any event is always positive. It could be at the least equal to zero but not less than that. The probability of sure event at the maximum could be equal to 1 so probability lies between 0 and 1 both included.

17.

(d)  $\frac{1}{2}$

**Explanation:**

$\frac{1}{2}$

18.

(d) 30 – 40

**Explanation:**

According to the question,

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Freq	3	9	15	30	18	5

Here Maximum frequency is 30.

Therefore, the modal class is 30 – 40.

19. (a) Both A and R are true and R is the correct explanation of A.

**Explanation:**

Both A and R are true and R is the correct explanation of A.

20. (a) Both A and R are true and R is the correct explanation of A.

**Explanation:**

Both A and R are true and R is the correct explanation of A.

### Section B

21. Given

$\sqrt{3}$  is an irrational number

Let  $5 + 2\sqrt{3}$  is a rational number

$\therefore$  we can write  $5 + 2\sqrt{3} = \frac{p}{q}$ , where p and q are integers

$$\Rightarrow 2\sqrt{3} = \frac{p}{q} - 5 = \frac{p-5q}{q}$$

$$\sqrt{3} = \frac{p-5q}{2q}$$

Here,  $\frac{p-5q}{2q}$  is a rational number

So,  $\sqrt{3}$  is also a rational number.

But it is given that  $\sqrt{3}$  is irrational number.

$\Rightarrow$  our assumption was wrong

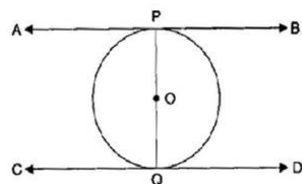
$\Rightarrow 5 + 2\sqrt{3}$  is an irrational number.

22.  $\angle ABD = \angle ACD \Rightarrow AB = AC$

$$\therefore \frac{BC}{BD} = \frac{BE}{AC} \Rightarrow \frac{BC}{BD} = \frac{BE}{BA}$$

$\angle B = \angle B$  (common)

$\therefore \triangle ABD \sim \triangle EBC$  [SAS similarity criterion]



23.

Given: PQ is a diameter of a circle with centre O.

The lines AB and CD are the tangents at P and Q respectively.

To Prove:  $AB \parallel CD$

Proof: Since AB is a tangent to the circle at P and OP is the radius through the point of contact.

$$\therefore \angle OPA = 90^\circ \dots\dots\dots (i)$$

[The tangent at any point of a circle is  $\perp$  to the radius through the point of contact]

$\therefore$  CD is a tangent to the circle at Q and OQ is the radius through the point of contact.

$$\therefore \angle OQD = 90^\circ \dots\dots\dots (ii)$$

[The tangent at any point of a circle is  $\perp$  to the radius through the point of contact]

From eq. (i) and (ii),  $\angle OPA = \angle OQD$

But these form a pair of equal alternate angles also,

$$\therefore AB \parallel CD$$

$$\begin{aligned} 24. & 2(\sin^2 45^\circ + \cot^2 30^\circ) - 6(\cos^2 45^\circ - \tan^2 30^\circ) \\ & \left[ \left( \frac{1}{\sqrt{2}} \right)^2 + (\sqrt{3})^2 \right] - 6 \left[ \left( \frac{1}{\sqrt{2}} \right)^2 - \left( \frac{1}{\sqrt{3}} \right)^2 \right] \\ & = 7 - 1 = 6 \end{aligned}$$

OR

$$\begin{aligned} \text{LHS} &= \frac{\cot A - \cos A}{\cot A + \cos A} \\ &= \frac{\frac{1}{\tan A} - \frac{1}{\sec A}}{\frac{1}{\tan A} + \frac{1}{\sec A}} \\ &= \frac{\frac{\sec A - \tan A}{\sec A \tan A}}{\frac{\sec A + \tan A}{\sec A \tan A}} \times \frac{\sec A - \tan A}{\sec A - \tan A} \\ &= \frac{(\sec^2 A - \tan^2 A)^2}{\sec^2 A - \tan^2 A} \\ &= \sec^2 A + \tan^2 A - 2 \sec A \tan A = \text{RHS} \end{aligned}$$

$$\begin{aligned} 25. \text{ Area of minor sector} &= \frac{3.14 \times (6)^2 \times 60^\circ}{360^\circ} \\ &= 18.84 \end{aligned}$$

Hence, area of minor sector is  $18.84 \text{ cm}^2$

Area of major sector = Area of circle - Area of minor sector

$$= 3.14 \times (6)^2 - 18.84$$

$$= 94.2$$

Hence, area of major sector is  $94.2 \text{ cm}^2$

OR

$$\text{Area of circle} = 3.14 \times 10 \times 10 = 314 \text{ cm}^2$$

$$\text{Area of minor sector} = \frac{3.14 \times 10 \times 10 \times 90}{360} = \frac{157}{2} \text{ cm}^2 \text{ or } 78.5 \text{ cm}^2$$

$$\text{Area of major sector} = 314 - 78.5 = 235.5 \text{ cm}^2$$

### Section C

26. The greatest number of cartons is the HCF of 144 and 90

Now the prime factorization of 144 and 90 are

$$144 = 16 \times 9 = 2^4 \times 3^2$$

$$90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$$

$$\text{HCF}(144, 90) = 2 \times 3^2 = 18$$

$\therefore$  The greatest number of cartons each stack would have = 18.

27. Let the given polynomial is  $p(x) = 4x^2 + 4x + 1$

Since,  $\alpha, \beta$  are zeroes of  $p(x)$ ,

$$\therefore \alpha + \beta = \text{sum of zeroes} = \frac{-4}{4}$$

$$\text{Also, } \alpha \cdot \beta = \text{Product of zeroes} = \alpha \cdot \beta = \frac{1}{4}$$

Now a quadratic polynomial whose zeroes are  $2\alpha$  and  $2\beta$

$$x^2 - (\text{sum of zeroes})x + \text{Product of zeroes}$$

$$= x^2 - (2\alpha + 2\beta)x + 2\alpha \times 2\beta$$

$$= x^2 - 2(\alpha + \beta)x + 4(\alpha\beta)$$

$$= x^2 - 2 \times (-1)x + 4 \times \frac{1}{4}$$



$$= x^2 + 2x + 1$$

The quadratic polynomial whose zeroes are  $2\alpha$  and  $2\beta$  is  $x^2 + 2x + 1$

28. The first 15 multiples of 8 are 8, 16, 24, 32,....

$$\text{Here, } a_2 - a_1 = 16 - 8 = 8$$

$$a_3 - a_2 = 24 - 16 = 8$$

$$a_4 - a_3 = 32 - 24 = 8$$

i.e.  $a_{k-1} - a_k$  is the same everytime.

So, the above list of numbers forms an AP.

$$\text{Here, } a = 8$$

$$d = 8$$

$$n = 15$$

$\therefore$  Sum of first 15 multiples of 8 =  $S_{15}$

$$= \frac{15}{2} [2a + (15 - 1)d] \dots \because S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$= \frac{15}{2} [2a + 14d]$$

$$= 15 (a + 7d)$$

$$= (15)(8 + 7 \times 8)$$

$$= (15) (8 + 56)$$

$$= (15) (64)$$

$$= 960$$

OR

Let  $a$  be the first term and the common difference be  $d$ .

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$S_{12} = \frac{12}{2} [2a + (12 - 1)d]$$

$$= 6[2a + 11d]$$

$$= 12a + 66d$$

$$S_8 = \frac{8}{2} [2a + (8 - 1)d]$$

$$= 4[2a + 7d]$$

$$= 8a + 28d$$

$$S_4 = \frac{4}{2} [2a + (4 - 1)d]$$

$$= 2[2a + 3d]$$

$$= 4a + 6d$$

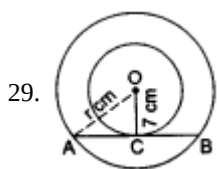
$$3(S_8 - S_4) = 3[(8a + 28d) - (4a + 6d)]$$

$$= 3[8a + 28d - 4a - 6d]$$

$$= 3[4a + 22d]$$

$$= 12a + 66d$$

$$= S_{12}$$



Let  $O$  be the common centre of the two circles and  $AB$  be the chord of the larger circle which touches the smaller circle at  $C$ .

Then,  $AB = 46$  cm.

Join  $OA$  and  $OC$

Then,  $OA = r$  cm and  $OC = 7$  cm.

Now,  $OC \perp AB$  and  $OC$  bisects  $AB$ . [as perpendicular to a chord through the chord bisects it]

In right,  $\triangle ACO$ , we have,

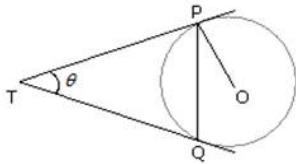
$$OA^2 = OC^2 + AC^2 \text{ [by Pythagoras' theorem]}$$

$$\Rightarrow OA = \sqrt{OC^2 + AC^2}$$

$$= \sqrt{OC^2 + \left(\frac{1}{2}AB\right)^2} \text{ [}\because C \text{ is the midpoint of } AB\text{]}$$

$$\begin{aligned}\Rightarrow r\text{cm} &= \sqrt{7^2 + 23^2}\text{cm} \\ &= \sqrt{578}\text{cm} \\ &= 17\sqrt{2} \\ \Rightarrow r &= 17\sqrt{2}\text{cm}.\end{aligned}$$

OR



Given A circle with centre O and an external point T and two tangents TP and TQ to the circle, where P, Q are the points of contact.

To Prove:  $\angle PTQ = 2\angle OPQ$

Proof: Let  $\angle PTQ = \theta$

Since TP, TQ are tangents drawn from point T to the circle.

$TP = TQ$

$\therefore$  TPQ is an isoscles triangle

$$\therefore \angle TPQ = \angle TQP = \frac{1}{2} (180^\circ - \theta) = 90^\circ - \frac{\theta}{2}$$

Since, TP is a tangent to the circle at point of contact P

$$\therefore \angle OPT = 90^\circ$$

$$\therefore \angle OPQ = \angle OPT - \angle TPQ = 90^\circ - (90^\circ - \frac{1}{2} \theta) = \frac{\theta}{2} = \frac{1}{2} \angle PTQ$$

Thus,  $\angle PTQ = 2\angle OPQ$

30. To prove :  $\frac{\tan \theta}{1 - \tan \theta} - \frac{\cot \theta}{1 - \cot \theta} = \frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}$

$$\begin{aligned}\text{L.H.S} &= \frac{\tan \theta}{1 - \tan \theta} - \frac{\cot \theta}{1 - \cot \theta} \\ &= \frac{\frac{\sin \theta}{\cos \theta}}{1 - \frac{\sin \theta}{\cos \theta}} - \frac{\frac{\cos \theta}{\sin \theta}}{1 - \frac{\cos \theta}{\sin \theta}} \dots [\because \tan \theta = \frac{\sin \theta}{\cos \theta}] \\ &= \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\cos \theta - \sin \theta}{\cos \theta}} + \frac{\frac{\cos \theta}{\sin \theta}}{\frac{\sin \theta - \cos \theta}{\sin \theta}} \\ &= \frac{\sin \theta + \cos \theta}{\cos \theta - \sin \theta} \\ &= \text{R.H.S.}\end{aligned}$$

31.

Class Interval	Frequency( $f_i$ )	Mid value $x_i$	$u_i = \frac{x_i - A}{h}$ $= \frac{x_i - 40}{10}$	$(f_i \times u_i)$
5 - 15	6	10	-3	-18
15 - 25	10	20	-2	-20
25 - 35	16	30	-1	-16
35 - 45	15	40 = A	0	0
45 - 55	24	50	1	24
55 - 65	8	60	2	16
65 - 75	7	70	3	21
	$\Sigma f_i = 86$			$\Sigma (f_i \times u_i) = 7$

Thus,  $A = 40$ ,  $h = 10$ ,  $\Sigma f_i = 86$  and  $\Sigma f_i u_i = 7$

$$\begin{aligned}\text{Mean} &= A + \left\{ h \times \frac{\Sigma f_i u_i}{\Sigma f_i} \right\} \\ &= 40 + \left\{ 10 \times \frac{7}{86} \right\} \\ &= 40 + 0.81 \\ &= 40.81\end{aligned}$$

#### Section D

32. Let breadth of the rectangular park = x m

Then, length of the rectangular park = (x + 3) m



Now, area of the rectangular park is  $= x(x + 3) = (x^2 + 3x)m^2$  [ $\because$  area = length  $\times$  breadth]

Given, base of the triangular park = Breadth of the rectangular park

Therefore, base of triangular park is  $= x$  m

altitude of triangular park is  $= 12$  m

Therefore, area of the triangular park will be  $= \frac{1}{2} \times x \times 12 = 6x m^2$  [ $\because$  area  $= \frac{1}{2} \times$  base  $\times$  height]

As per the question area of rectangular park is  $= 4 +$  Area of triangular park

$$\Rightarrow x^2 + 3x = 4 + 6x$$

$$\Rightarrow x^2 + 3x - 6x - 4 = 0$$

$$\Rightarrow x^2 - 3x - 4 = 0$$

$$\Rightarrow x^2 - 4x + x - 4 = 0 \text{ [by factorization]}$$

$$\Rightarrow x(x - 4) + 1(x - 4) = 0$$

$$\Rightarrow (x - 4)(x + 1) = 0$$

$$\Rightarrow x - 4 = 0 \text{ or } x + 1 = 0$$

$$\Rightarrow x = 4 \text{ or } x = -1$$

Since, breadth cannot be negative, so we will neglect  $x = -1$  and choose  $x = 4$

Hence, breadth of the rectangular park will be  $= 4$  m

and length of the rectangular park will be  $= x + 3 = 4 + 3 = 7$  m

OR

Given,

$$(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$$

$$\Rightarrow x^2 - ax - bx + ab + x^2 - bx - cx + bc + x^2 - cx - ax + ac = 0$$

$$\Rightarrow 3x^2 - 2ax - 2bx - 2cx + ab + bc + ca = 0$$

For equal roots  $B^2 - 4AC = 0$

$$\text{or, } \{-2(a + b + c)\}^2 = 4 \times 3(ab + bc + ca)$$

$$\text{or, } 4(a + b + c)^2 - 12(ab + bc + ca) = 0$$

$$\text{or, } a^2 + b^2 + c^2 + 2ab + 2bc + 2ac - 3ab - 3bc - 3ac = 0$$

$$\text{or, } \frac{1}{2}[2a^2 + 2b^2 + 2c^2 - 2ab - 2ac - 2bc] = 0$$

$$\text{or, } \frac{1}{2}[(a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ac)] = 0$$

$$\text{or, } \frac{1}{2}[(a^2 + b^2 - 2ab) + (b^2 + c^2 - 2bc) + (c^2 + a^2 - 2ac)] = 0$$

$$\text{or, } (a - b)^2 + (b - c)^2 + (c - a)^2 = 0 \text{ if } a \neq b \neq c$$

$$\text{Since } (a - b)^2 > 0, (b - c)^2 > 0, (c - a)^2 > 0$$

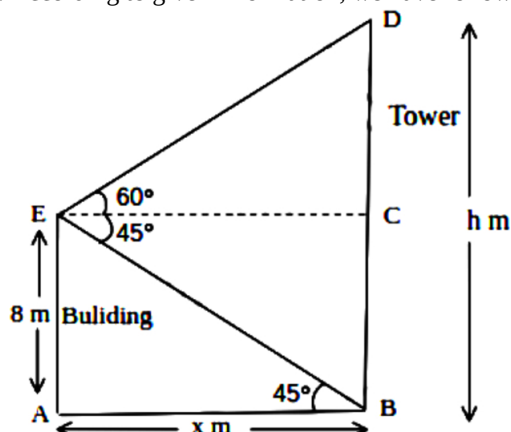
$$\text{Hence, } (a - b)^2 = 0 \Rightarrow a = b$$

$$(a - c)^2 = 0 \Rightarrow b = c$$

$$(c - a)^2 = 0 \Rightarrow c = a$$

$\therefore a = b = c$  Hence Proved.

33. According to given information, we have following figure



Let  $BD = h$  m and  $AB = x$  m

Then,  $CD = h - 8$  ... ( $\because BC = 8$  m)

Also, In  $\triangle EAB$ , we have

$$\tan 45^\circ = \frac{8}{x}$$

$$\Rightarrow 1 = \frac{8}{x}$$

$$\Rightarrow x = 8 \text{ m}$$

Similarly, In  $\triangle DCE$ , we have

$$\tan 60^\circ = \frac{CD}{EC}$$

$$\Rightarrow \sqrt{3} = \frac{h-8}{x} \dots (\because EC = AB = x \text{ m})$$

$$\Rightarrow \sqrt{3}x = h - 8$$

$$\Rightarrow 8\sqrt{3} = h - 8 \dots (\because x = 8 \text{ m})$$

$$\Rightarrow h = 8(\sqrt{3} + 1)$$

$$= 8 \times 2.732$$

$$= 21.856 \text{ m}$$

34. We have;

A Cube,

Cube's  $\frac{\text{length}}{\text{Edge}}$ ,  $a = 7 \text{ cm}$

A Cylinder:

Cylinder's Radius,  $r = 2.1 \text{ cm}$  or  $r = \frac{21}{10} \text{ cm}$

Cylinder's Height,  $h = 7 \text{ cm}$

$\therefore$  A cylinder is scooped out from a cube,

$\therefore$  TSA of the resulting cuboid:

= TSA of whole Cube -  $2 \times$  (Area of upper circle or Area of lower circle) + CSA of the scooped out Cylinder

$$= 6a^2 + 2\pi rh - 2 \times (\pi r^2)$$

$$= 6 \times (7)^2 + 2 \times (22 \div 7 \times 2.1 \times 7) - 2 \times [22 \div 7 \times (2.1)^2]$$

$$= 6 \times 49 + (44 \div 7 \times 14.7) - (44 \div 7 \times 4.41)$$

$$= 294 + 92.4 - 27.72$$

$$= 294 + 64.68$$

$$= 358.68 \text{ cm}^2$$

Hence, the total surface area of the remaining solid is  $358.68 \text{ cm}^2$

OR

Height of the cylinder = 3 m.

Total height of the tent above the ground = 13.5 m

height of the cone =  $(13.5 - 3) \text{ m} = 10.5 \text{ m}$

Radius of the cylinder = radius of cone = 14 m

$$\text{Curved surface area of the cylinder} = 2\pi rh \text{ m}^2 = \left(2 \times \frac{22}{7} \times 14 \times 3\right) \text{ m}^2 = 264 \text{ m}^2$$

$$\therefore l = \sqrt{r^2 + h^2} = \sqrt{14^2 + (10.5)^2} = \sqrt{196 + 110.25} = \sqrt{306.25} = 17.5$$

$$\therefore \text{Cured surface area of the cone} = \pi rl = \left(\frac{22}{7} \times 14 \times 17.5\right) \text{ m}^2 = 770 \text{ m}^2$$

Let S be the total area which is to be painted. Then,

$S = \text{Curved surface area of the cylinder} + \text{Curved surface area of the cone}$

$$\Rightarrow S = (264 + 770) \text{ m}^2 = 1034 \text{ m}^2$$

Hence, Cost of painting =  $S \times \text{Rate} = ₹ (1034 \times 2) = ₹ 2068$

35. Table:

Class	Frequency	Mid value $x_i$	$f_i x_i$	Cumulative frequency
25 - 35	7	30	210	7
35 - 45	31	40	1240	38
45 - 55	33	50	1650	71
55 - 65	17	60	1020	88
65 - 75	11	70	770	99
75 - 85	1	80	80	100
	N = 100		$\sum f_i x_i = 4970$	



i. Mean

$$\frac{\sum f_i x_i}{\sum f_i} = \frac{4970}{100} = 49.70$$

ii.  $N = 100$ ,  $\frac{N}{2} = 50$

Median Class is 45 - 55

$$l = 45, h = 10, N = 100, c = 38, f = 33$$

$$\therefore \text{Median} = l + h \left( \frac{\frac{N}{2} - c}{f} \right)$$

$$= 45 + \left\{ 10 \times \frac{50 - 38}{33} \right\}$$

$$= 45 + 3.64 = 48.64$$

iii. we know that,  $\text{Mode} = 3 \times \text{median} - 2 \times \text{mean}$

$$= 3 \times 48.64 - 2 \times 49.70$$

$$= 145.92 - 99.4 = 46.52$$

### Section E

36. i. Let the fixed charge be ₹ x and per kilometer charge be ₹ y

$$\therefore x + 10y = 105 \dots(i)$$

$$x + 15y = 155 \dots(ii)$$

From (i) and (ii)

$$5y = 50$$

$$y = \frac{50}{5} = 10$$

From equation (i)

$$x + 100 = 105$$

$$x = 105 - 100 = 5$$

Fixed charges = ₹ 5

ii. Let the fixed charge be ₹ x and per kilometer charge be ₹ y

$$\therefore x + 10y = 105 \dots(1)$$

$$x + 15y = 155 \dots(2)$$

From (1) and (2)

$$5y = 50$$

$$y = \frac{50}{5} = 10$$

From equation (1)

$$x + 100 = 105$$

$$x = 105 - 100 = 5$$

Per km charges = ₹ 10

iii. Let the fixed charge be ₹ a and per kilometer charge be ₹ b

$$a + 10b$$

$$20 + 10 \times 10 = ₹ 120$$

**OR**

$$\text{Total amount} = x + 10y + x + 25y$$

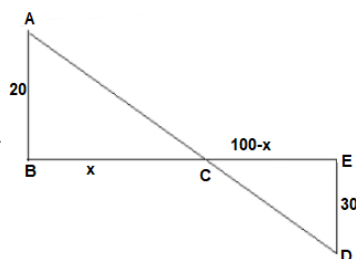
$$= 2x + 35y$$

$$= 2 \times 5 + 35 \times 10$$

$$= 10 + 350$$

$$= ₹ 360$$

37. i.



$$\triangle ABC \sim \triangle DEC$$

$$\frac{20}{30} = \frac{x}{100-x}$$

$$2000 - 20x = 30x$$

$$2000 = 50x$$

$$x = 40 \text{ m}$$

ii. AA

iii. 60 metres

**OR**

$$AD = AC + CD$$

$$= \sqrt{20^2 + 40^2} + \sqrt{60^2 + 30^2}$$

$$= \sqrt{400 + 1600} + \sqrt{3600 + 900}$$

$$= \sqrt{2000} + \sqrt{4500}$$

$$\Rightarrow 20\sqrt{5} + 30\sqrt{5}$$

$$\Rightarrow 50\sqrt{5} \text{ m}$$

38. i. (4, 8) and (-3, 7)

ii. 8 units

iii. 1280 cubic feet

**OR**

7 or -1

